
Analog and Digital Signals and Systems

R.K. RAO YARLAGADDA

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*This book is dedicated to my wife
Marceil, children, Tammy Bardwell, Ryan Yarlagadda
and Travis Yarlagadda and their families*

Note to Instructors

The solutions manual can be located on the book's webpage [http://www/springer.com/engineering/cirucits+ %26+systems/bok/978-1-4419-0033-3](http://www.springer.com/engineering/cirucits+%26+systems/bok/978-1-4419-0033-3)

Preface

This book presents a systematic, comprehensive treatment of analog and discrete signal analysis and synthesis and an introduction to analog communication theory. This evolved from my 40 years of teaching at Oklahoma State University (OSU). It is based on three courses, Signal Analysis (a second semester junior level course), Active Filters (a first semester senior level course), and Digital signal processing (a second semester senior level course). I have taught these courses a number of times using this material along with existing texts. The references for the books and journals (over 160 references) are listed in the bibliography section. At the undergraduate level, most signal analysis courses do not require probability theory. Only, a very small portion of this topic is included here.

I emphasized the basics in the book with simple mathematics and the sophistication is minimal. Theorem-proof type of material is not emphasized. The book uses the following model:

1. Learn *basics*
2. Check the work using *bench marks*
3. Use *software* to see if the results are accurate

The book provides detailed *examples* (over 400) with applications. A three-number system is used consisting of chapter number – section number – example or problem number, thus allowing the student to quickly identify the related material in the appropriate section of the book. The book includes well *over 400 homework problems*. Problem numbers are identified using the above three-number system. Hints are provided wherever additional details may be needed and may not have been given in the main part of the text. A detailed *solution manual* will be available from the publisher for the instructors.

Summary of the Chapters

This book starts with an *introductory chapter* that includes most of the basic material that a junior in electrical engineering had in the beginning classes. For those who have forgotten, or have not seen the material recently, it gives enough

background to follow the text. The topics in this chapter include singularity functions, periodic functions, and others. Chapter 2 deals with convolution and correlation of periodic and aperiodic functions. Chapter 3 deals with approximating a function by using a set of basis functions, referred to as the generalized Fourier series expansion. From these concepts, the three basic Fourier series expansions are derived. The discussion includes detailed discussion on the operational properties of the Fourier series and their convergence.

Chapter 4 deals with Fourier transform theory derived from the Fourier series. Fourier series and transforms are the bases to this text. Considerable material in the book is based on these topics. Chapter 5 deals with the relatives of the Fourier transforms, including Laplace, cosine and sine, Hartley and Hilbert transforms.

Chapter 6 deals with basic systems analysis that includes linear time-invariant systems, stability concepts, impulse response, transfer functions, linear and nonlinear systems, and very simple filter circuits and concepts. Chapter 7 starts with the Bode plots and later deals with approximations using classical analog Butterworth, Chebyshev, and Bessel filter functions. Design techniques, based on both amplitude and phase based, are discussed. Last part of this chapter deals with analysis and synthesis of active filter circuits. Examples of basic low-pass, high-pass, band-pass, band elimination, and delay line filters are included.

Chapter 8 builds a bridge to go from the continuous-time to discrete-time analysis by starting with sampling theory and the Fourier transform of the ideally sampled signals. Bulk of this chapter deals with discrete basis functions, discrete-time Fourier series, discrete-time Fourier transform (DTFT), and the discrete Fourier transform (DFT). Chapter 9 deals with fast implementations of the DFT, discrete convolution, and correlation. Second part of the chapter deals the z -transforms and their use in the design of discrete-data systems. Digital filter designs based on impulse invariance and bilinear transformations are presented. The chapter ends with digital filter realizations.

Chapter 10 presents an introduction to analog communication theory, which includes basic material on analog modulation, such as AM and FM, demodulation, and multiplexing. Pulse modulation methods are introduced.

Appendix A reviews the basics on matrices; Appendix B gives a brief introduction on MATLAB; and Appendix C gives a list of useful formulae. The book concludes with a list of references and Author and Subject indexes.

Suggested Course Content

Instructor is the final judge of what topics will best suit his or her class and in what depth. The suggestions given below are intended to serve as a guide only. The book permits flexibility in teaching analysis, synthesis of continuous-time and discrete-time systems, analog filters, digital signal processing, and an introduction to analog communications. The following table gives suggestions for courses.

	Topical Title	Related topics in chapters
One semester	(Fundamentals of analog signals and systems)	Chapters 1–4, 6
One semester	Systems and analog filters	Chapters 4, 5*, 6, 7
One semester	(Introduction to digital signal processing)	Chapters 4*, 6*, 8, 9
Two semesters	(Signals and an introduction to analog communications)	Chapters 1–4, 5*, 6, 8*, 10

*Partial coverage

Acknowledgements

The process of writing this book has taken me several years. I am indebted to all the students who have studied with me and taken classes from me. Education is a two-way street. The teachers learn from the students, as well as the students learn from the teachers. Writing a book is a learning process.

Dr. Jack Cartinhour went through the material in the early stages of the text and helped me in completing the solution manual. His suggestions made the text better. I am deeply indebted to him. Dr. George Scheets used an earlier version of this book in his signal analysis and communications theory class. Dr. Martin Hagan has reviewed a chapter. Their comments were incorporated into the manuscript. Beau Lacefield did most of the artwork in the manuscript. Vijay Venkataraman and Wen Fung Leong have gone through some of the chapters and their suggestions have been incorporated. In addition, Vijay and Wen have provided some of the MATLAB programs and artwork. I appreciated Vijay's help in formatting the final version of the manuscript.

An old adage of the uncertainty principle is, no matter how many times the author goes through the text, mistakes will remain. I sincerely appreciate all the support provided by Springer. Thanks to Alex Greene. He believed in me to complete this project. I appreciated the patience and support of Katie Chen. Thanks to Shanty Jaganathan and her associates of Integra-India. They have been helpful and gracious in the editorial process.

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Finally, the time spent on this book is the time taken away from my wife Marceil, children Tammy, Ryan and Travis and my grandchildren. Without my family's understanding, I could not have completed this book.

Oklahoma, USA

R.K. Rao Yarlagadda

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